A Unified Stochastic Physics Framework for Simulating Uncertainty in Subgrid Processes

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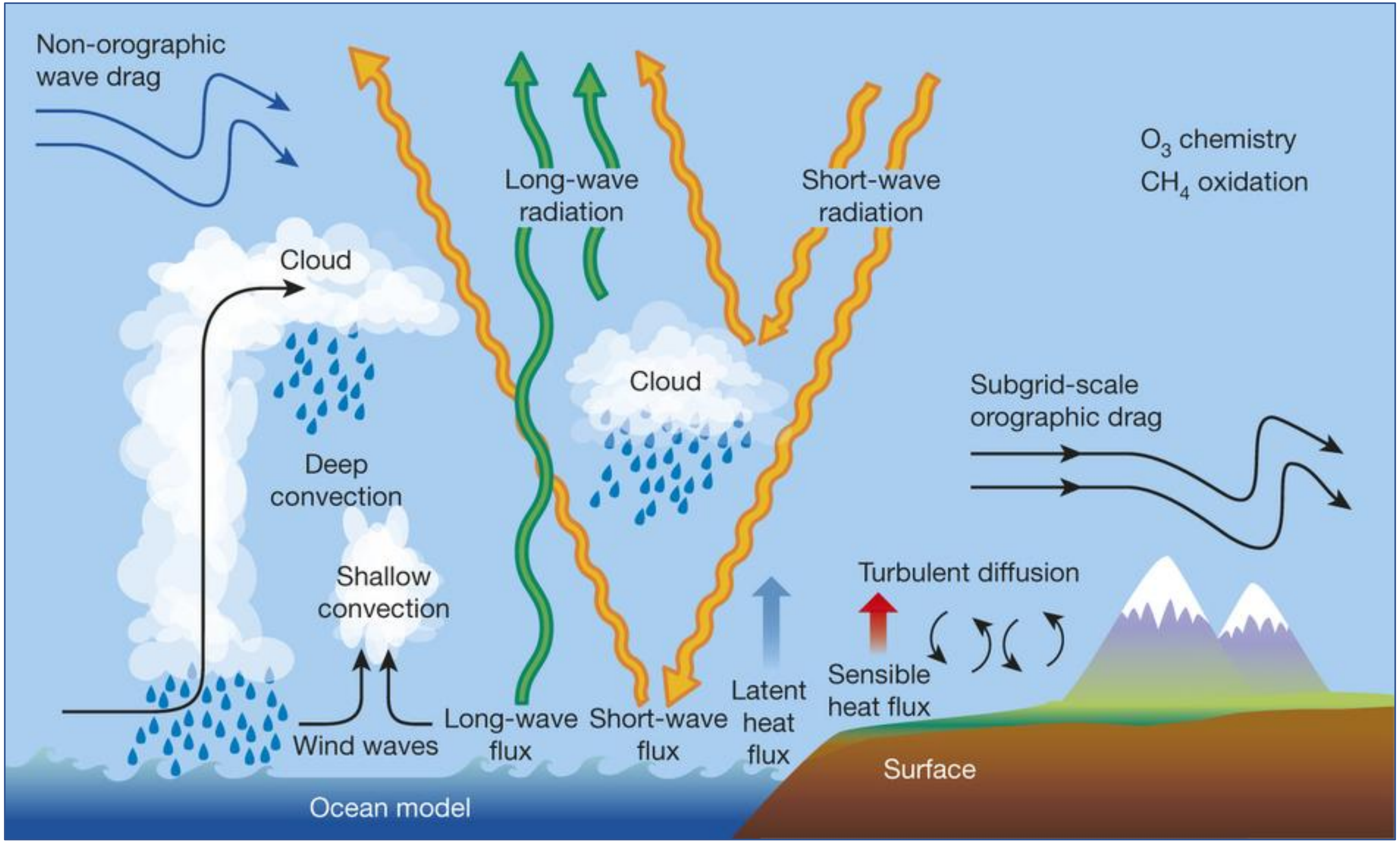
Major uncertainty in NOAA's UFS

Subgrid physical processes represented via parameterizations describing their contributions to the resolved scales in terms of mass, momentum and heat transfers



Peter Bauer et al. (2015)

How do we represent uncertainty in NOAA's operational models?

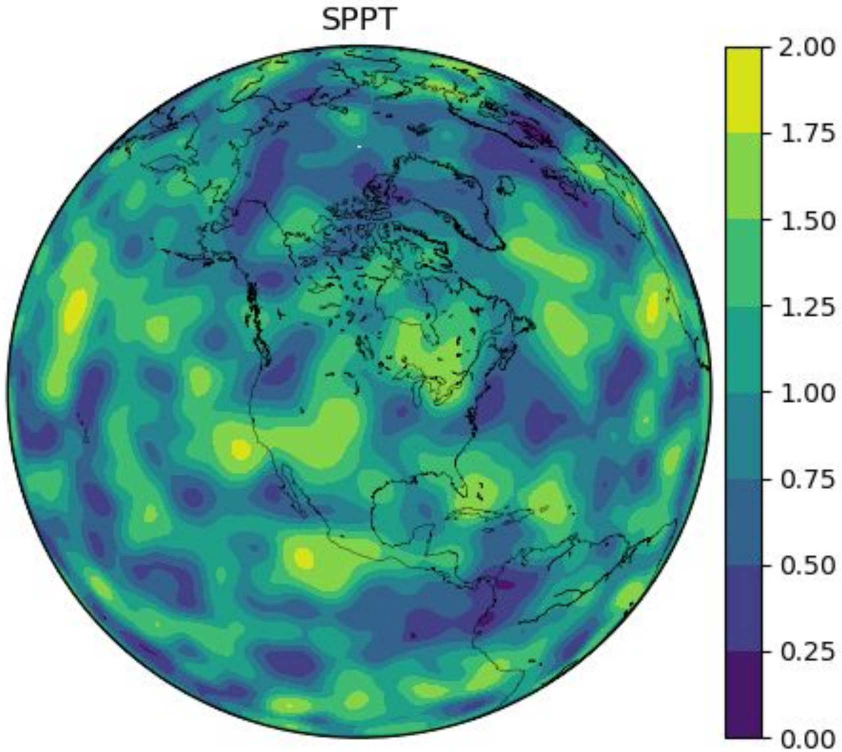
- Stochastically Perturbed Physics Tendencies (SPPT) scheme: simulates uncertainty due to sub-grid physics parameterizations (Palmer et al., 2009)
- Stochastic Kinetic Energy Backscatter Scheme (SKEB): simulated the uncertainty in turbulent energy cascade (Palmer et al., 2009)
- Stochastically-perturbed boundary-layer humidity (SHUM) scheme: perturbs boundary layer humidity following Tompkins and Berner (2008)
- VC scheme: vorticity confinement based on Sanchez et al (2012)

All use stochastic random pattern generators to generate spatially and temporally correlated noise.

An example: the SPPT scheme

$$Total\ Physics\ Tendency = (1+\hat{e})\sum_{i=1}^{N}\left(Individual\ Physics\ Tendecy\right)_i$$

$(1+\hat{e})$



N = 5 in the GEFS:

1. Radiation
2. Surface fluxes
3. Turbulent mixing and gravity wave drag
4. Convection
5. Microphysics

Physics tendencies of four variables are randomly perturbed:
U, V, T, q

The random perturbation, $\hat{e}$, is invariant vertically but tapered in the boundary layer and stratosphere

(adapted from ECMWF)

The Mori-Zwanzig formalism: A general principle for representing subgrid uncertainty

- The model system can generally be expressed in the phase space as the following:

$$\dot{\boldsymbol{x}} = \boldsymbol{M}(\boldsymbol{x}(t)),\ \boldsymbol{x}(0) = \boldsymbol{x}_0,\ \boldsymbol{x} = \overline{\boldsymbol{x}} + \widetilde{\boldsymbol{x}},$$

  where $\overline{\boldsymbol{x}}$ and $\widetilde{\boldsymbol{x}}$ denote resolved and unresolved model state, respectively.

- Rewrite model as the so-called Liouville equation:

$$\partial \boldsymbol{z}/\partial t = \boldsymbol{L}\boldsymbol{z}(\boldsymbol{x}, t),\ \boldsymbol{z}(\boldsymbol{x}, 0) = \boldsymbol{a}(\boldsymbol{x}),$$

  where the Liouville operator, $\boldsymbol{L}$, is defined as

$$\boldsymbol{L} = \boldsymbol{M} \cdot \nabla$$

(Chorin et al., Optimal prediction and the Mori-Zwanzig representation of irreversible processes, PANS, 2000)

The Mori-Zwanzig formalism (cont'd)

- The following generalized Langevin equation can be obtained by using the Mori-Zwanzig projection operators to map the Liouville equation on to the resolved and sub-grid variables

$$\dot{\boldsymbol{x}} = \boldsymbol{e}^{t\boldsymbol{L}}\boldsymbol{P}\boldsymbol{L}\overline{\boldsymbol{x}}_0 + \int_0^t \boldsymbol{e}^{(t-s)\boldsymbol{L}}\boldsymbol{P}\boldsymbol{L}\boldsymbol{e}^{s\boldsymbol{Q}\boldsymbol{L}}\boldsymbol{Q}\boldsymbol{L}\overline{\boldsymbol{x}}_0 ds + \boldsymbol{e}^{t\boldsymbol{Q}\boldsymbol{L}}\boldsymbol{Q}\boldsymbol{L}\,\overline{\boldsymbol{x}}_0$$

Resolved dynamics

"memory" term because it is an integration of quantities that are dependent on the model state at earlier times

"noise" term, representing the unresolved dynamics

where $\boldsymbol{P}$ is the projection to map $\boldsymbol{z}(\boldsymbol{x}, t)$ onto the resolved variables and $\boldsymbol{Q} = \boldsymbol{1} - \boldsymbol{P}$ is the projection to map $\boldsymbol{z}(\boldsymbol{x}, t)$ onto the subgrid variables

(Chorin et al., Optimal prediction and the Mori-Zwanzig representation of irreversible processes, PANS, 2000)

The multidimensional Langevin process (MLP)

In the physics literature, stochastic processes described by the generalized Langevin equation are called multi-dimensional Langevin Processes (MLP). Two approaches have been pursued to reduce the stochastic simulation of model uncertainty from the generalized Langevin equation to either (1) autoregressive models, AR(q) or (2) autoregressive moving average models, ARMA(q, p). Thus, the minimal form of the MLP for model uncertainty simulation is the following AR(1) process

$$\delta \boldsymbol{x}(t + \Delta t) = \phi \delta \boldsymbol{x}(t) + \rho \eta(t) \Delta t [d\delta \boldsymbol{x}(t)/dt]_{physics} \,,$$

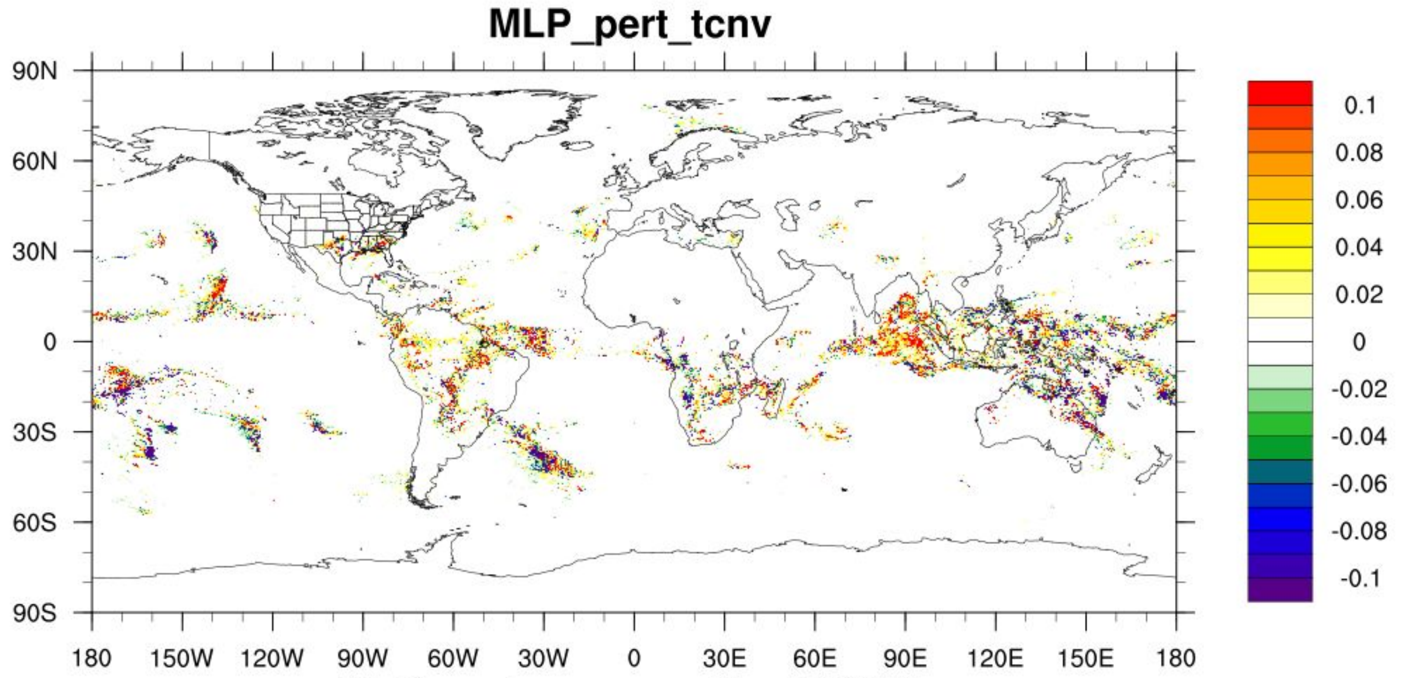
which is a dimensional analogy to the currently widely-used stochastic random pattern generators

$$\hat{e}(t + \Delta t) = \phi \, \hat{e}(t) + \rho \eta(t)$$

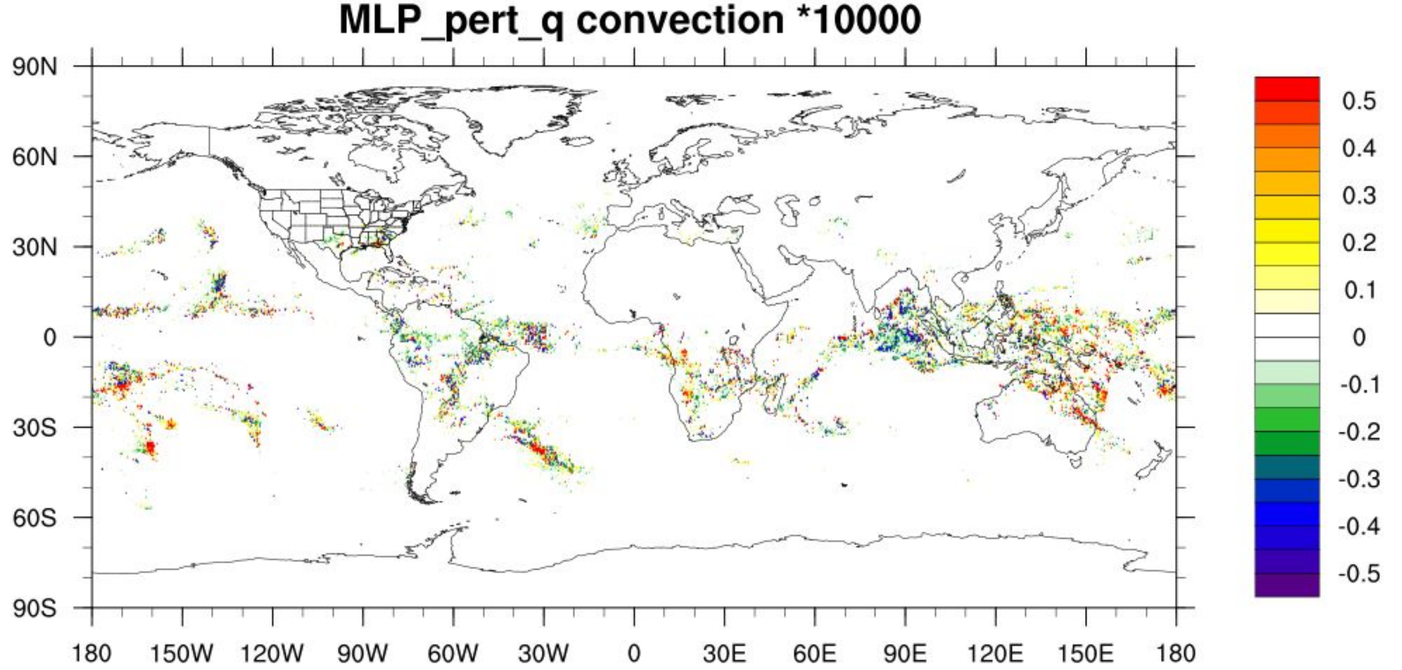
(See, e.g., Shutts, A stochastic convective backscatter scheme for use in ensemble prediction systems, *Q.J.R. Met. Soc.*, 2015)

Perturbations of temperature and moisture from the convection scheme at 500 mb at Forecast time 36h

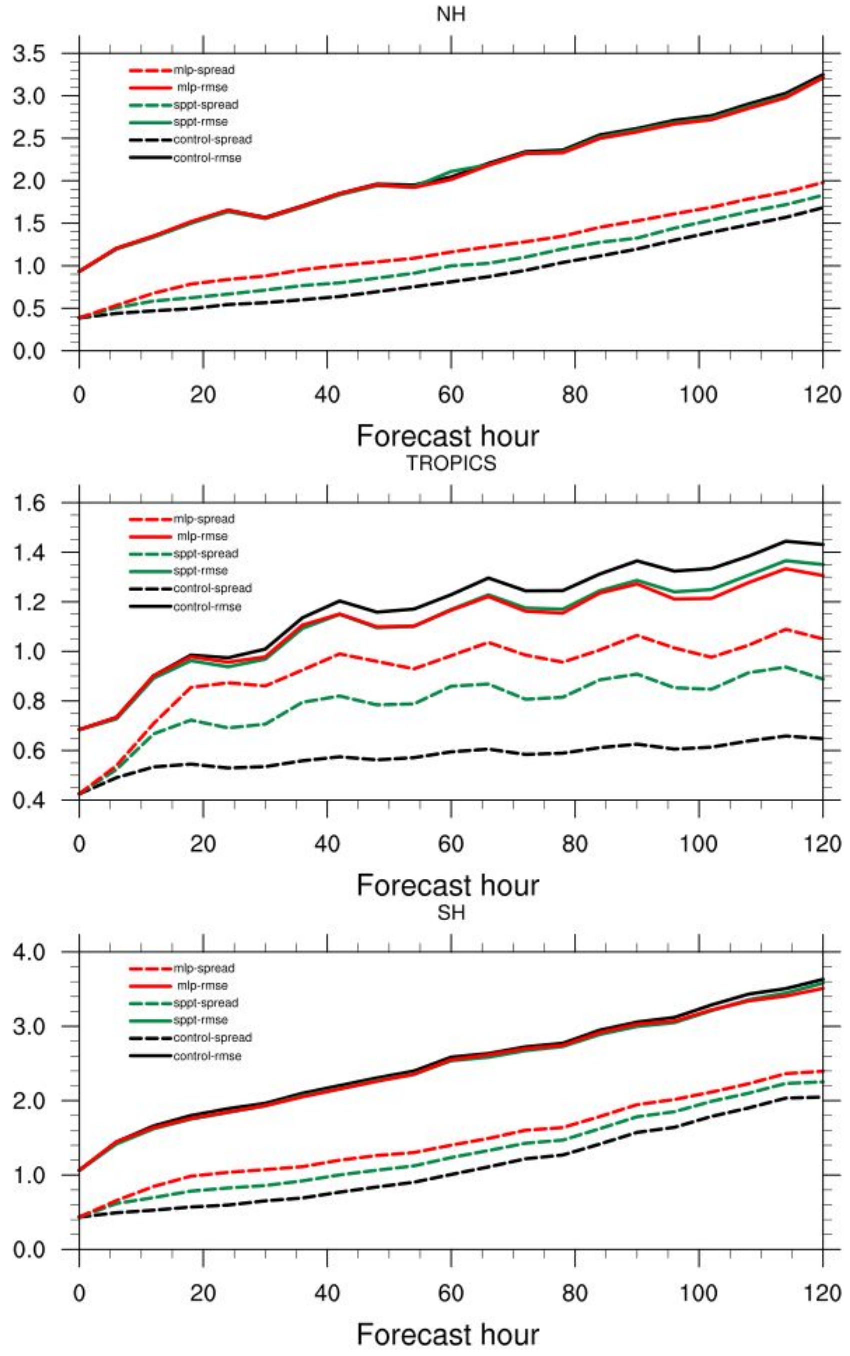
δT_{conv}



δq_{conv}



Solid=RMSE
Dashed=Spread



850 mb Temperature RMSE and Spread averaged over 18 cases initialized at 0000 UTC on: January 1, January 15, April 1, April 15, July 1, July 15, October 1, October 15, and November 1 for both 2016 and 2017. Top panel is averaged over the Northern Hemisphere, the middle panel is averaged over the tropics and the lower panel is averaged over the Southern Hemisphere.

Concluding remarks

- The basis of mathematical physics for stochastic physics parameterizations can be built upon the Mori-Zwanzig formalism.

- Up to this point in our ongoing development, the importance of the memory term, as often discussed in the literature of statistical physics, has rarely been explicitly stated in the discussion of the stochastic physics development for NWP.

- Because our development is based on a general theory of model error simulation/modeling, the outcome of the development is a unified stochastic physics parameterization framework applicable to all UFS applications to deal with the complexity of model uncertainty quantification and simulation.